

ERRATA AND COMMENTS

“Discrete Control Systems”

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The following ‘red’ letters are errata that should be corrected (or inserted).

Chapter 1 Mathematical Descriptions and Models

- 1 Page 5 Eq. (1.10),

$$y(k+n) + \cdots + a_{n-1}y(k+1) + a_n y(k) = b_0 u(k+n) + \cdots + b_{n-1}u(k+1) + b_n u(k)$$

- 2 Page 6 Eq. (1.18),

$$y(k) = [b_n - a_n b_0 \quad b_{n-1} - a_{n-1} b_0 \quad \cdots \quad b_1 - a_1 b_0] \begin{bmatrix} x_1(k) \\ x_2(k) \\ \vdots \\ x_n(k) \end{bmatrix} + b_0 u(k).$$

- 3 Fig. 1.5 Time sequences of the solution for Example 1.6

- 4 Below Fig. 1.5:

Note that the response is delayed by one step as shown in Fig. 1.5 if $y(k+1) = x_1(k)$ (i.e., $y(k) = x_1(k-1)$) is applied to the computer program for (1.55). This response corresponds to the result of Exercise (7).

- 5 Page 7 Eq. (1.21),

$$x(k) = Ax(k-1) + Bu(k-1),$$

- 6 Page 13 Eq. (1.35),

$$y(k+n) + \cdots + a_{n-1}y(k+1) + a_n y(k) = b_0 u(k+n) + \cdots + b_{n-1}u(k+1) + b_n u(k).$$

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$$\begin{cases} \mathcal{Z}\{y(k+n)\} = z^n \hat{y}(z) - (y(0)z^n + \dots + y(n-1)z) \\ \dots \\ \mathcal{Z}\{a_{n-1}y(k+1)\} = a_{n-1}(z\hat{y}(z) - y(0)z) \\ \mathcal{Z}\{a_n y(k)\} = a_n \hat{y}(z). \end{cases}$$

8 Page 13 For simplicity, the initial conditions are assumed to be zero (i.e., $y(0) = y(1) = \dots = y(n-1) = 0$ and also $u(0) = u(1) = \dots = u(n-1) = 0$).

Comments:

There would be no contradiction, because $y(\kappa)$ and $u(\kappa)$ in (1.35) defined for $\kappa = k+n$ ($\kappa \geq n$). However, in a computer simulation, backward expressions (1.19), (1.21), and (1.40) should be used.

9 Page 13 Eq. (1.36),

$$(z^n + a_1 z^{n-1} + \dots + a_{n-1} z + a_n) \hat{y}(z) = (b_0 z^n + b_1 z^{n-1} + \dots + b_{n-1} z + b_n) \hat{u}(z).$$

10 Page 14 Eq. (1.38), The z -transform with respect to $\kappa = k+2$ is given as

$$(z^2 - z + 0.5) \hat{y}(z) - y(0)z^2 - y(1)z + y(0)z = (z+1) \hat{u}(z) - u(0)z.$$

and

$$(z^2 - z + 0.5) \hat{y}(z) - y(0)z^2 - y(1)z + y(0)z = \frac{z(z+1)}{z-1} - u(0)z$$

11 Page 18

$$\frac{z^2 + z}{z^3 - 2z^2 + 1.5z - 0.5} \Rightarrow \frac{z^2 + z}{z^3 - 2z^2 + 1.5z - 0.5}$$

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$$\begin{cases} \begin{bmatrix} x_1(k+1) \\ x_2(k+1) \end{bmatrix} = \begin{bmatrix} -a_1 & 1 \\ -a_2 & 0 \end{bmatrix} \begin{bmatrix} x_1(k) \\ x_2(k) \end{bmatrix} + \begin{bmatrix} b_1 \\ b_2 \end{bmatrix} u(k), \\ y(k) = x_1(k), \text{ where } a_1 = -1, a_2 = 0.5, \text{ and } b_1 = b_2 = 1 \end{cases}$$

Tips:

The z -transforms of the above equations are given as:

$$\begin{bmatrix} z-1 & -1 \\ 0.5 & z \end{bmatrix} \begin{bmatrix} \hat{x}_1(z) \\ \hat{x}_2(z) \end{bmatrix} = \begin{bmatrix} \hat{u}(z) \\ \hat{u}(z) \end{bmatrix}$$

Then,

$$\begin{bmatrix} \hat{x}_1(z) \\ \hat{x}_2(z) \end{bmatrix} = \frac{1}{z^2 - z + 0.5} \begin{bmatrix} z & 1 \\ 0.5 & z-1 \end{bmatrix} \begin{bmatrix} z/(z-1) \\ z/(z-1) \end{bmatrix}.$$

Thus,

$$\hat{y}(z) = \hat{x}_1(z) = \frac{z^2 + z}{(z^2 - z + 0.5)(z - 1)} = \frac{z^2 + z}{z^3 - 2z^2 + 1.5z - 0.5}.$$

13 [Page 19](#) The caption in Fig. 1.6,

Fig. 1.6 Block diagram for Example 1.6, where $a_1 = 1$, $a_2 = -0.5$, and $b_1 = b_2 = 1$

14 [Page 23](#), Eq. (1.68),

$$G_1(z) := \tilde{\mathcal{Z}}\{G_1(s)\} = \frac{K_0}{1 - z^{-1}} + \frac{K_1}{1 - e^{p_1 h} z^{-1}} + \cdots + \frac{K_n}{1 - e^{p_n h} z^{-1}}.$$

15 [Page 24](#) in Table 1.2,

The third and fourth lines in ‘Discrete time’,

$$e^{p_k h} \quad \Leftrightarrow \quad k h e^{p_k h}$$

16 [Page 25](#) Eq. (1.76),

$$\Phi(\tau) := e^{A\tau} = \mathbf{I} + A\tau + \frac{A^2 \tau^2}{2!} + \cdots,$$

where

$$\mathbf{I} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

17 [Page 26](#) Eqs. (1.78), (1.79), and (1.80)),

My first manuscript was written as follows:

$$\begin{cases} \mathbf{x}(k+1) = \Phi(h)\mathbf{x}(k) + \Gamma(h)u(k), & \Gamma(h) = \int_0^h \Phi(\tau)\mathbf{B}d\tau \\ y(k) = \mathbf{C}\mathbf{x}(k). \end{cases}$$

$$\begin{cases} z\hat{\mathbf{x}}(z) = \Phi\hat{\mathbf{x}}(z) + \Gamma\hat{u}(z) \\ \hat{y}(z) = \mathbf{C}\hat{\mathbf{x}}(z). \end{cases}$$

$$\hat{y}(z) = \mathbf{C}[\mathbf{I} - \Phi z^{-1}]^{-1} \Gamma z^{-1} \hat{u}(z).$$

These expressions might be preferable to (1.78), (1.79), and (1.80).

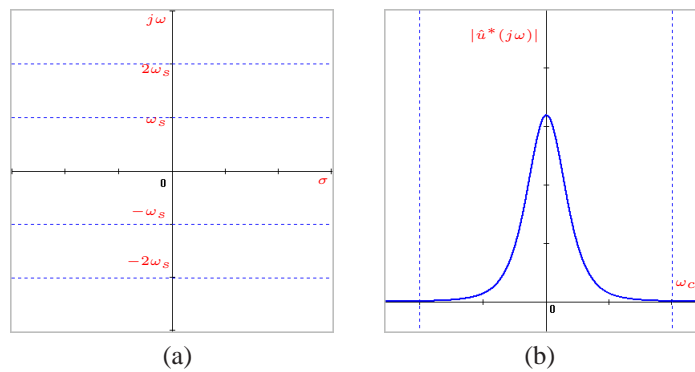


Fig. 1.23 Frequency shifting and spectrum.

Chapter 2 Discretized Feedback Systems

- 1 Page 50 Eq. (2.7)

$$-\beta e^2 \leq g(e)e \leq \beta e^2.$$

- 2 Page 56 In Eq. (2.25):

$$e^{*\dagger}(z) \Rightarrow \hat{e}^{*\dagger}(z)$$

- 3 Page 69 Eq. (2.62)

$$\sum_{k,l=1,k \neq l}^N |x(k)|^2 |y(l)|^2 - 2 \sum_{k,l=1,k \neq l}^N |x(k)y(k)| \cdot |x(l)y(l)| = \sum_{k,l=1,k \neq l}^N |x(k)y(l) - x(l)y(k)|^2,$$

- 4 Page 71

In the last line

... and the inequality problem is proved.

Chapter 3 Robust Stability Analysis

- 1 Page 75 Eq. (3.8)

$$\|e(k)\|_2 \leq \|r(k)\|_2 + \sup_{|z|=1} |G(z)| (\rho \cdot \|e(k)\|_2 + \|d(k)\|_2).$$

- 2 **Page 94** Eq. (3.46) in Theorem 3.3,

$$\eta(q_0, \omega_0) = \max_q \min_\omega \eta(q, \omega)$$

- 3 **Page 94**

The verification of robust stability using the above modified Hall diagram (off-axis M -circles) is based on the following theorem.

Chapter 4 Model Reference Feedback and PID Control

- 1 **Page 114** in Eqs. (4.16), (4.17), (4.20), and (4.21),

$$\begin{aligned} G_I(s) &\Rightarrow G_1(s) \\ G_I(z) &\Rightarrow G_1(z). \end{aligned}$$

- 2 **Page 117**

In Eq. (4.22),

$$\hat{f}(z) \Rightarrow \mathcal{F}(z).$$

- 3 **Page 117**

In the first line under Fig. 12,

....., characteristic equation **of the nominal feedback system** is given as

- 4 **Page 132**

$$\tilde{y}_3^{(3)} = \tilde{y}_3^{(2)} - \frac{a_{32}^{(2)}}{a_{22}^{(2)}} \tilde{y}_2^{(2)}, \quad \Rightarrow \quad \tilde{y}_3^{(3)} = \tilde{y}_3^{(1)} - \frac{a_{31}^{(1)}}{a_{11}^{(1)}} \tilde{y}_1^{(1)} - \frac{a_{32}^{(2)}}{a_{22}^{(2)}} \tilde{y}_2^{(2)},$$

- 5 **Page 140**

In Exercise (3), determine the characteristic equation **of the nominal system**, $\mathcal{F}(z) = 0$, for Example 4.1 (A)

- 6 **Page 140**

(4) Show that the approximate PID control system in Fig. 4.18 is obtained from the model-reference feedback system in Fig. 4.17, when $\mathcal{D}_m(\cdot)$ and $\mathcal{D}_f(\cdot)$, are **in high resolution**.

Chapter 5 Multi-Loop Feedback Systems

- 1 **Page 153**

... the right side of (5.13) ... \Rightarrow the right side of (5.17) ...

$$\tilde{y}_n^{(n)} = \tilde{y}_n^{(n-1)} - \frac{a_{n,n-1}^{(n-1)}}{a_{n-1,n-1}} y_{n-1}^{(n-1)} \Rightarrow \tilde{y}_n^{(n)} = \tilde{y}_n^{(1)} - \frac{a_{n1}^{(1)}}{a_{11}^{(1)}} \tilde{y}_1^{(1)} - \frac{a_{n2}^{(2)}}{a_{22}^{(2)}} \tilde{y}_2^{(2)} - \dots - \frac{a_{n,n-1}^{(n-1)}}{a_{n-1,n-1}} \tilde{y}_{n-1}^{(n-1)}$$

2 Page 153

$$\dots, \text{ where } y_j^{(j)} \Rightarrow \dots, \text{ where } 0 < \tilde{y}_j^{(j)} \leq y_j^{(j)}$$

3 Page 153

... all principal minors of matrix \mathcal{A} \Rightarrow ... all principal minors of matrix (5.15)

4 Page 163

$$a_{11}^{(1)} > 0, a_{22}^{(2)} > 0, \dots, a_{nn}^{(n)} \Rightarrow a_{11}^{(1)} > 0, a_{22}^{(2)} > 0, \dots, a_{nn}^{(n)} > 0$$

$$\tilde{y}_n^{(n)} = \tilde{y}_n^{(n-1)} - \frac{a_{n,n-1}^{(n-1)}}{a_{n-1,n-1}} y_{n-1}^{(n-1)} \Rightarrow \tilde{y}_n^{(n)} = \tilde{y}_n^{(1)} - \frac{a_{n1}^{(1)}}{a_{11}^{(1)}} \tilde{y}_1^{(1)} - \frac{a_{n2}^{(2)}}{a_{22}^{(2)}} \tilde{y}_2^{(2)} - \dots - \frac{a_{n,n-1}^{(n-1)}}{a_{n-1,n-1}} \tilde{y}_{n-1}^{(n-1)}$$

5 Page 163

$$\dots, \text{ where } y_j^{(j)} \Rightarrow \dots, \text{ where } 0 < \tilde{y}_j^{(j)} \leq y_j^{(j)}$$

6 Page 163

... all principal minors of matrix \mathcal{A} \Rightarrow ... all principal minors of matrix (5.48)

7 Page 177

$$y_j \geq 0, J = 1, 2, \dots, n \text{ and } a_{ij} \leq 0, i \neq j \\ \Rightarrow \tilde{y}_j \geq 0, j = 1, 2, \dots, n \text{ and } a_{ij} \leq 0, i \neq j$$

Chapter 6 Interval Polynomials and Robust Performance

1 Page 186 Equation (6.17) should be written as follows:

$$\begin{aligned} \tilde{F}(z) = & D_{c1}(z)D_{c2}(z)D_{11}(z)D_{22}(z)D_{12}(z)D_{21}(z) \\ & + [K_1^-, K_1^+]N_{c1}(z)N_{11}(z)D_{c2}(z)D_{22}(z)D_{12}(z)D_{21}(z) \\ & + [K_2^-, K_2^+]N_{c2}(z)N_{22}(z)D_{c1}(z)D_{11}(z)D_{12}(z)D_{21}(z) \\ & + [K^-, K^+]N_{c1}(z)N_{c2}(z)N_{11}(z)N_{22}(z)D_{12}(z)D_{21}(z) \\ & - [K^-, K^+]N_{c1}(z)N_{c2}(z)N_{12}(z)N_{21}(z)D_{11}(z)D_{22}(z) = 0. \end{aligned}$$

2 Page 191

Fig. 6.3 ... for discrete control system \Rightarrow Fig. 6.3 ... for discrete control systems

3 Page 192

$$\phi = \tan^{-1} \left(\frac{-\gamma + 2(1 + \gamma^2)\theta - \gamma(1 + \gamma^2)\theta^2}{1 - (1 + \gamma^2)\theta^2} \right)$$

The proof of Lemma 6.1 is given as follows:

$$\begin{aligned} x^2 + (y - \gamma)^2 &= \frac{[1 - (1 + \gamma^2)\theta^2]^2 + [-\gamma + 2\theta(1 + \gamma^2) - \gamma(1 + \gamma^2)\theta^2]^2}{[1 - 2\gamma\theta + (1 + \gamma^2)\theta^2]^2} \\ &= \frac{(1 + \gamma^2)[1 + (1 + \gamma^2)\theta^4 - 2\theta^2 + 4(1 + \gamma^2)\theta^2 + \gamma^2(1 + \gamma^2)\theta^4 - 4\gamma\theta - 4\gamma(1 + \gamma^2)\theta^3 + 2\gamma^2\theta^2]}{[1 - 2\gamma\theta + (1 + \gamma^2)\theta^2]^2} \\ &= \frac{(1 + \gamma^2)[1 + 4\gamma^2\theta^2 + (1 + \gamma^2)^2\theta^4 - 4\gamma\theta - 4\gamma(1 + \gamma^2)\theta^3 + 2(1 + \gamma^2)\theta^2]}{[1 - 2\gamma\theta + (1 + \gamma^2)\theta^2]^2} = 1 + \gamma^2. \end{aligned}$$

Thus, Lemma 6.1 has been proved. \square

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$$\tilde{F}(s) = [a_0^-, a_0^+]s^3 + [a_1^-, a_1^+]s^2 + [a_2^-, a_2^+]s + [a_3^-, a_3^+]$$

Chapter 7 Relation to Discrete Event Systems

1 Page 228 Fig. 7.6 Petri net systems \Rightarrow Fig. 7.6 Petri net systems

(In the following figures, transitions τ_i are written in t_i)

2 Page 232 In (7.21),

$$\forall x_0 \in S(\mathcal{X}_m; r) \Rightarrow \forall x_0 \in S(\mathcal{X}_m; r)$$

3 Page 234

Their notation is ... \Rightarrow The notations are ...

4 Page 234

$$\|x_i(t_k)\|_{\ell_1} = \sum_{k=0}^{\infty} |x_i(t_k)|$$

and

$$\|\mathbf{x}(t_k)\|_{\ell_1} = \begin{bmatrix} \|x_1(t_k)\|_{\ell_1} \\ \|x_2(t_k)\|_{\ell_1} \\ \vdots \\ \|x_n(t_k)\|_{\ell_1} \end{bmatrix}.$$

5 Page 235 In (7.28) and (7.33),

$$\Psi(t_k) \Rightarrow \Psi(t_k)$$

6 Page 236

$$I - \left(\sum_{l=1}^k |\Phi(t_k, t_l)| \right) \bar{\psi} \Rightarrow I - \left(\sum_{l=1}^k |\Phi(t_k, t_l)| \right) \bar{\Psi}$$

7 Page 237

... and three events. \Rightarrow ... and two events.

8 Page 239

$$\|\mathbf{x}(t_k)\|_{\ell_p} = \begin{bmatrix} \|x_1(t_k)\|_{\ell_p} \\ \|x_2(t_k)\|_{\ell_p} \\ \vdots \\ \|x_n(t_k)\|_{\ell_p} \end{bmatrix}$$

9 Page 239 The equations for continuous-time systems should be corrected as follows:

$$\|x_i(\tau)\|_{L_p} = \left(\int_0^\infty |x_i(\tau)|^p d\tau \right)^{1/p}.$$

and

$$\|\mathbf{x}(\tau)\|_{L_p} = \begin{bmatrix} \|x_1(\tau)\|_{L_p} \\ \|x_2(\tau)\|_{L_p} \\ \vdots \\ \|x_n(\tau)\|_{L_p} \end{bmatrix},$$

furthermore,

$$\|\Psi(\tau)\|_{L_1} = \begin{bmatrix} \int_0^\infty |\psi_{11}(\tau)| d\tau & \int_0^\infty |\psi_{12}(\tau)| d\tau & \dots & \int_0^\infty |\psi_{1n}(\tau)| d\tau \\ \int_0^\infty |\psi_{21}(\tau)| d\tau & \int_0^\infty |\psi_{22}(\tau)| d\tau & \dots & \int_0^\infty |\psi_{2n}(\tau)| d\tau \\ \vdots & \vdots & \ddots & \vdots \\ \int_0^\infty |\psi_{n1}(\tau)| d\tau & \int_0^\infty |\psi_{n2}(\tau)| d\tau & \dots & \int_0^\infty |\psi_{nn}(\tau)| d\tau \end{bmatrix}.$$

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Four discrete-type equation \Rightarrow Forward discrete-time equation