ERRATA AND COMMENTS

"Discrete Control Systems"

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The following 'red' letters are errata that should be corrected (or inserted).

Chapter 1 Mathematical Descriptions and Models

1 Page 5 Eq. (1.10),

$$y(k+n)+\cdots+a_{n-1}y(k+1)+a_n y(k)=b_0u(k+n)+\cdots+b_{n-1}u(k+1)+b_n u(k)$$

2 Page 6 Eq. (1.18),

$$y(k)=\begin{bmatrix} b_n-a_nb_0 & b_{n-1}-a_{n-1}b_0 & \ldots & b_1-a_1b_0 \end{bmatrix}\begin{bmatrix} x_1(k) \\ x_2(k) \\ \vdots \\ x_n(k) \end{bmatrix}+b_0u(k).$$

3 Fig. 1.5 Time sequences of the solution for Example 1.6

4 Below Fig. 1.5:
Note that the response is delayed by one step as shown in Fig. 1.5 if $y(k+1)=x_1(k)$ (i.e., $y(k)=x_1(k-1)$) is applied to the computer program for (1.55). This response corresponds to the result of Exercise (7).

5 Page 7 Eq. (1.21),

$$\boldsymbol{x}(k)=\boldsymbol{A}\boldsymbol{x}(k-1)+\boldsymbol{B}u(k-1),$$

6 Page 13 Eq. (1.35),

$$y(k+n)+\cdots+a_{n-1}y(k+1)+a_n y(k)=b_0u(k+n)+\cdots+b_{n-1}u(k+1)+b_n u(k).$$

7 Page 13

$$\begin{cases} \mathcal{Z}\{y(k+n)\} = z^n\hat{y}(z) - (y(0)z^n + \cdots + y(n-1)z) \\ \qquad \cdots \\ \mathcal{Z}\{a_{n-1}y(k+1)\} = a_{n-1}(z\hat{y}(z) - y(0)z) \\ \mathcal{Z}\{a_n y(k)\} = a_n\hat{y}(z). \end{cases}$$

8 Page 13 For simplicity, the initial conditions are assumed to be zero (i.e., $y(0) = y(1) = \cdots = y(n-1) = 0$ and also $u(0) = u(1) = \cdots = u(n-1) = 0$).

Comments:

There would be no contradiction, because $y(\kappa)$ and $u(\kappa)$ in (1.35) defined for $\kappa = k+n$ $(\kappa \geq n)$. However, in a computer simulation, backward expressions (1.19), (1.21), and (1.40) should be used.

9 Page 13 Eq. (1.36),

$$(z^n + a_1 z^{n-1} + \cdots + a_{n-1}z + a_n)\hat{y}(z) = (b_0 z^n + b_1 z^{n-1} + \cdots + b_{n-1}z + b_n)\hat{u}(z).$$

10 Page 14 Eq. (1.38), The z-transform with respect to $\kappa = k+2$ is given as

$$(z^2 - z + 0.5)\hat{y}(z) - y(0)z^2 - y(1)z + y(0)z = (z+1)\hat{u}(z) - u(0)z.$$

and

$$(z^2 - z + 0.5)\hat{y}(z) - y(0)z^2 - y(1)z + y(0)z = \frac{z(z+1)}{z-1} - u(0)z$$

11 Page 18

$$\frac{z^2 + z)}{z^3 - 2z^2 + 1.5z - 0.5} \quad \Rightarrow \quad \frac{z^2 + z}{z^3 - 2z^2 + 1.5z - 0.5}$$

12 Page 18

$$\begin{cases} \begin{bmatrix} x_1(k+1) \\ x_2(k+1) \end{bmatrix} = \begin{bmatrix} -a_1 & 1 \\ -a_2 & 0 \end{bmatrix} \begin{bmatrix} x_1(k) \\ x_2(k) \end{bmatrix} + \begin{bmatrix} b_1 \\ b_2 \end{bmatrix} u(k), \\ y(k) = x_1(k), \quad \text{where } a_1 = -1, \ a_2 = 0.5, \quad \text{and } b_1 = b_2 = 1 \end{cases}$$

Tips:

The z-transforms of the above equations are given as:

$$\begin{bmatrix} z-1 & -1 \\ 0.5 & z \end{bmatrix} \begin{bmatrix} \hat{x}_1(z) \\ \hat{x}_2(z) \end{bmatrix} = \begin{bmatrix} \hat{u}(z) \\ \hat{u}(z) \end{bmatrix}$$

Then,

$$\begin{bmatrix} \hat{x}_1(z) \\ \hat{x}_2(z) \end{bmatrix} = \frac{1}{z^2 - z + 0.5} \begin{bmatrix} z & 1 \\ 0.5 & z-1 \end{bmatrix} \begin{bmatrix} z/(z-1) \\ z/(z-1) \end{bmatrix}.$$

Thus,

$$\hat{y}(z) = \hat{x}_1(z) = \frac{z^2+z}{(z^2) - z + 0.5)(z-1)} = \frac{z^2+z}{z^3 - 2z^2 + 1.5z - 0.5}.$$

13 Page 19 The caption in Fig. 1.6,

Fig. 1.6 Block diagram for Example 1.6, where $a_1 = 1$, $a_2 = -0.5$, and $b_1 = b_2 = 1$

14 Page 23, Eq. (1.68),

$$G_1(z) := \tilde{\mathcal{Z}}\{G_1(s)\} = \frac{K_0}{1 - z^{-1}} + \frac{K_1}{1 - \mathrm{e}^{p_1 h} z^{-1}} + \cdots + \frac{K_n}{1 - \mathrm{e}^{p_n h} z^{-1}}.$$

15 Page 24 in Table 1.2,
The fouth line in 'Discrete time',

$$\mathrm{e}^{p_k h} \quad \Rightarrow \quad kh\mathrm{e}^{p_k h}$$

16 Page 25 Eq. (1.76),

$$\boldsymbol{\Phi}(\tau) := \mathrm{e}^{\boldsymbol{A}\tau} = \boldsymbol{I} + \boldsymbol{A}\tau + \frac{\boldsymbol{A}^2\tau^2}{2!} + \cdots,$$

where

$$\boldsymbol{I} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

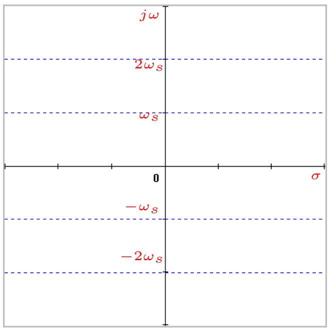
17 Page 26 Eqs. (1.78), (1.79), and (1.80)),
My first manuscript was written as follows:

$$\begin{cases} \boldsymbol{x}(k+1) = \boldsymbol{\Phi}(h)\boldsymbol{x}(k) + \boldsymbol{\Gamma}(h)u(k), \quad \boldsymbol{\Gamma}(h) = \int_0^h \boldsymbol{\Phi}(\tau)\boldsymbol{B}\mathrm{d}\tau \\ y(k) = \boldsymbol{C}\boldsymbol{x}(k). \end{cases}$$

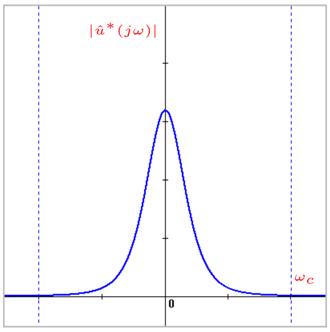
$$\begin{cases} z\hat{\boldsymbol{x}}(z) = \boldsymbol{\Phi}\hat{\boldsymbol{x}}(z) + \boldsymbol{\Gamma}\hat{u}(z) \\ \hat{y}(z) = \boldsymbol{C}\hat{\boldsymbol{x}}(z). \end{cases}$$

$$\hat{y}(z) = \boldsymbol{C}[\boldsymbol{I} - \boldsymbol{\Phi}z^{-1}]^{-1}\boldsymbol{\Gamma}z^{-1}\hat{u}(z).$$

These expressions might be preferable to (1.78), (1.79), and (1.80).



(a)



(b)

Fig. 1 .23 Frequency shifting and spectrum.

Chapter 2 Discretized Feedback Systems

1 Page 50 Eq. (2.7)

$$-\beta e^2 \leq g(e)e \leq \beta e^2.$$

2 Page 56 In Eq. (2.25):

$$e^{*\dagger}(z) \quad \Rightarrow \quad \hat{e}^{*\dagger}(z)$$

3 Page 69 Eq. (2.62)

$$\sum_{k,l=1,k\neq l}^{N} |x(k)|^2 |y(l)|^2 - 2 \sum_{k,l=1,k\neq l}^{N} |x(k)y(k)| \cdot |x(l)y(l)| = \sum_{k,l=1,k\neq l}^{N} |x(k)y(l) - x(l)y(k)|^2,$$

4 Page 71

In the last line

$\cdots$ and the inequality problem is proved.

Chapter 3 Robust Stability Analysis

1 Page 75 Eq. (3.8)

$$\|e(k)\|_2 \leq \|r(k)\|_2 + \sup_{|z|=1} |G(z)|(\rho \cdot \|e(k)\|_2 + \|d(k)\|_2).$$

2 Page 94 Eq. (3.46) in Theorem 3.3,

$$\eta(q_0, \omega_0) - \max_{q} \min_{\omega} \eta(q, \omega)$$

3 Page 94

The verification of robust stability using the above modified Hall diagram (off-axis *M*-circles) is based on the following theorem.

Chapter 4 Model Reference Feedback and PID Control

1 Page 114 in Eqs. (4.16), (4.17), (4.20), and (4.21),

$$G_I(s) \Rightarrow G_1(s)$$
$$G_I(z) \Rightarrow G_1(z).$$

2 Page 117

In Eq. (4.22),

$$\hat{f}(z) \Rightarrow \mathcal{F}(z).$$

3 Page 117

In the first line under Fig. 12,

$\cdots\cdots$, characteristic equation of the nominal feedback system is given as

4 Page 132

$$\tilde{y}_3^{(3)} = \tilde{y}_3^{(2)} - \frac{a_{32}^{(2)}}{a_{22}^{(2)}}\tilde{y}_2^{(2)}, \quad \Rightarrow \quad \tilde{y}_3^{(3)} = \tilde{y}_3^{(1)} - \frac{a_{31}^{(1)}}{a_{11}^{(1)}}\tilde{y}_1^{(1)} - \frac{a_{32}^{(2)}}{a_{22}^{(2)}}\tilde{y}_2^{(2)},$$

5 Page 140

In Exercise (3), determine the characteristic equation of the nominal system, $\mathcal{F}(z) = 0$, for Example 4.1 (A) $\cdots\cdots$.

6 Page 140

(4) Show that the approximate PID control system in Fig. 4.18 is obtained from the model-reference feedback system in Fig. 4.17, when $\mathcal{D}_m(\cdot)$ and $\mathcal{D}_f(\cdot)$, are in high resolution.

Chapter 5 Multi-Loop Feedback Systems

1 Page 153

$$\cdots \text{ the right side of (5.13)} \cdots \Rightarrow \text{the right side of (5.17)} \cdots$$

$$\tilde{y}_n^{(n)} = \tilde{y}_n^{(n-1)} - \frac{a_{n,n-1}^{(n-1)}}{a_{n-1,n-1}^{(n-1)}} y_{n-1}^{(n-1)} \Rightarrow \tilde{y}_n^{(n)} = \tilde{y}_n^{(1)} - \frac{a_{n1}^{(1)}}{a_{11}^{(1)}} \tilde{y}_1^{(1)} - \frac{a_{n2}^{(2)}}{a_{22}^{(2)}} \tilde{y}_2^{(2)} - \cdots - \frac{a_{n,n-1}^{(n-1)}}{a_{n-1,n-1}^{(n-1)}} \tilde{y}_{n-1}^{(n-1)}$$

2 Page 153

$$\cdots, \text{ where } y_j^{(j)} \Rightarrow \cdots, \text{ where } 0 < \tilde{y}_j^{(j)} \leq y_j^{(j)}$$

3 Page 153

$$\cdots \text{ all principal minors of matrix } \mathcal{A} \Rightarrow \cdots \text{ all principal minors of matrix (5.15)}$$

4 Page 163

$$a_{11}^{(1)} > 0, a_{22}^{(2)} > 0, \cdots, a_{nn}^{((n)} \Rightarrow a_{11}^{(1)} > 0, a_{22}^{(2)} > 0, \cdots, a_{nn}^{((n)} > 0$$

$$\tilde{y}_n^{(n)} = \tilde{y}_n^{(n-1)} - \frac{a_{n,n-1}^{(n-1)}}{a_{n-1,n-1}^{(n-1)}} y_{n-1}^{(n-1)} \Rightarrow \tilde{y}_n^{(n)} = \tilde{y}_n^{(1)} - \frac{a_{n1}^{(1)}}{a_{11}^{(1)}} \tilde{y}_1^{(1)} - \frac{a_{n2}^{(2)}}{a_{22}^{(2)}} \tilde{y}_2^{(2)} - \cdots - \frac{a_{n,n-1}^{(n-1)}}{a_{n-1,n-1}^{(n-1)}} \tilde{y}_{n-1}^{(n-1)}$$

5 Page 163

$$\cdots, \text{ where } y_j^{(j)} \Rightarrow \cdots, \text{ where } 0 < \tilde{y}_j^{(j)} \leq y_j^{(j)}$$

6 Page 163

$$\cdots \text{ all principal minors of matrix } \mathcal{A} \Rightarrow \cdots \text{ all principal minors of matrix (5.48)}$$

7 Page 177

$$\begin{aligned} & y_j \geq 0, J = 1, 2, \cdots, n \text{ and } a_{ij} \leq 0, i \neq j \\ & \qquad \Rightarrow \tilde{y}_j \geq 0, j = 1, 2, \cdots, n \text{ and } a_{ij} \leq 0, i \neq j \end{aligned}$$

Chapter 6 Interval Polynomials and Robust Performance

1 Page 186 Equation (6.17) should be written as follows:

$$\begin{aligned} & \tilde{F}(z) = D_{c1}(z)D_{c2}(z)D_{11}(z)D_{22}(z)D_{12}(z)D_{21}(z) \\ & +[K_1^-, K_1^+]N_{c1}(z)N_{11}(z)D_{c2}(z)D_{22}(z)D_{12}(z)D_{21}(z) \\ & +[K_2^-, K_2^+]N_{c2}(z)N_{22}(z)D_{c1}(z)D_{11}(z)D_{12}(z)D_{21}(z) \\ & +[K^-, K^+]N_{c1}(z)N_{c2}(z)N_{11}(z)N_{22}(z)D_{12}(z)D_{21}(z) \\ & -[K^-, K^+]N_{c1}(z)N_{c2}(z)N_{12}(z)N_{21}(z)D_{11}(z)D_{22}(z) = 0. \end{aligned}$$

2 Page 191

Fig. 6.3 $\cdots$ for discrete control system $\Rightarrow$ Fig. 6.3 $\cdots$ for discrete control systems

3 Page 192

$$\phi = \tan^{-1}\left(\frac{-\gamma+2(1+\gamma^2)\theta - \gamma(1+\gamma^2)\theta^2}{1-(1+\gamma^2)\theta^2}\right)$$

The proof of Lemma 6.1 is given as follows:

$$x^2 + (y-\gamma)^2 = \frac{[1-(1+\gamma^2)\theta^2]^2 + [-\gamma + 2\theta(1+\gamma^2) - \gamma(1+\gamma^2)\theta^2]^2}{[1-2\gamma\theta + (1+\gamma^2)\theta^2]^2}$$

$$= \frac{(1+\gamma^2)[1+(1+\gamma^2)\theta^4 - 2\theta^2 + 4(1+\gamma^2)\theta^2 + \gamma^2(1+\gamma^2)\theta^4 - 4\gamma\theta - 4\gamma(1+\gamma^2)\theta^3 + 2\gamma^2\theta^2]}{[1-2\gamma\theta + (1+\gamma^2)\theta^2]^2}$$

$$= \frac{(1+\gamma^2)[1+4\gamma^2\theta^2 + (1+\gamma^2)^2\theta^4 - 4\gamma\theta - 4\gamma(1+\gamma^2)\theta^3 + 2(1+\gamma^2)\theta^2]}{[1-2\gamma\theta + (1+\gamma^2)\theta^2]^2} = 1+\gamma^2.$$

Thus, Lemma 6.1 has been proved. $\square$

4 Page 204

$$\tilde{F}(s) = [a_0^-, a_0^+]s^3 + [a_1^-, a_1^+]s^2 + [a_2^-, a_2^+]s + [a_3^-, a_3^+]$$

Chapter 7 Relation to Discrete Event Systems

1 Page 228 **Fig. 7.6** Petri net systems $\Rightarrow$ **Fig. 7.6** Petri net systems
(In the following figures, transitions τ_i are written in t_i)

2 Page 232 In (7.21),

$$\forall x0 \in S(\mathcal{X}_m; r) \;\Rightarrow\; \forall x_0 \in S(\mathcal{X}_m; r)$$

3 Page 234

Their notation is $\cdots$ $\Rightarrow$ The notations are $\cdots$

4 Page 234

$$\|x_i(t_k)\|_{\ell_1} = \sum_{k=0}^{\infty} |x_i(t_k)|$$

and

$$\|\boldsymbol{x}(t_k)\|_{\ell_1} = \begin{bmatrix} \|x_1(t_k)\|_{\ell_1} \\ \|x_2(t_k)\|_{\ell_1} \\ \vdots \\ \|x_n(t_k)\|_{\ell_1} \end{bmatrix}.$$

5 Page 235 In (7.28) and (7.33),

$$\boldsymbol{\Psi}(t_k)) \Rightarrow \boldsymbol{\Psi(t_k)}$$

6 Page 236

$$\boldsymbol{I} - \left(\sum_{l=1}^{k} |\boldsymbol{\Phi}(t_k, t_l)|\right) \bar{\boldsymbol{\psi}} \Rightarrow \boldsymbol{I} - \left(\sum_{l=1}^{k} |\boldsymbol{\Phi}(t_k, t_l)|\right) \bar{\boldsymbol{\Psi}}$$

7 Page 237

$\cdots$ and three events. $\Rightarrow \cdots$ and two events.

8 Page 239

$$\|\boldsymbol{x}(t_k)\|_{\ell_p} = \begin{bmatrix} \|x_1(t_k)\|_{\ell_p} \\ \|x_2(t_k)\|_{\ell_p} \\ \vdots \\ \|x_n(t_k)\|_{\ell_p} \end{bmatrix}$$

9 Page 239 The equations for continuous-time systems should be corrected as follows:

$$\|x_i(\tau)\|_{L_p} = \left(\int_0^\infty |x_i(\tau)|^p \mathrm{d}\tau\right)^{1/p}.$$

and

$$\|\boldsymbol{x}(\tau)\|_{L_p} = \begin{bmatrix} \|x_1(\tau)\|_{L_p} \\ \|x_2(\tau)\|_{L_p} \\ \vdots \\ \|x_n(\tau)\|_{L_p} \end{bmatrix},$$

furthermore,

$$\|\boldsymbol{\Psi}(\tau)\|_{L_1} = \begin{bmatrix} \int_0^\infty |\psi_{11}(\tau)|\mathrm{d}\tau & \int_0^\infty |\psi_{12}(\tau)|\mathrm{d}\tau & \ldots & \int_0^\infty |\psi_{1n}(\tau)|\mathrm{d}\tau \\ \int_0^\infty |\psi_{21}(\tau)|\mathrm{d}\tau & \int_0^\infty |\psi_{22}(\tau)|\mathrm{d}\tau & \ldots & \int_0^\infty |\psi_{2n}(\tau)|\mathrm{d}\tau \\ \vdots & \vdots & \ddots & \vdots \\ \int_0^\infty |\psi_{n1}(\tau)|\mathrm{d}\tau & \int_0^\infty |\psi_{n2}(\tau)|\mathrm{d}\tau & \ldots & \int_0^\infty |\psi_{nn}(\tau)|\mathrm{d}\tau \end{bmatrix}.$$

Index

1 Page 242

Four discrete-type equation $\Rightarrow$ Forward discrete-time equation