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Stability Analysis of Discrete Event Control Systems Based on Connection Matrices and Graphs

Yoshifumi Okuyama

Abstract— **There are many finite-state and event-driven types of discrete systems, e.g., manufacturing processes, industry and welfare robots, and networked control systems, and so on. However, the analysis and design of such discrete control systems have not been established, because those systems have severe nonlinear characteristics and do not respond continuously in time. In this paper, the stability of discrete event control systems is studied based on multiple metrics and simultaneous inequalities. Especially, in this paper, the structures of discrete event types of systems are analyzed using connection matrices (i.e., permutation (0,1)-matrices). The relationship between connection matrices and graph representations is also reviewed in general. A stability condition is derived based on the concept of nonnegative inverse matrices (so-called M-matrices). Numerical examples were shown to clarify the stability and boundedness of discrete-event control systems.**

I. Introduction

There are many finite state and event-driven types of discrete systems, e.g., manufacturing systems, industry and welfare robots, computer networks, and so on. However, the analysis and design of such discrete dynamic systems have not been established [1], [2], [3], because those systems have severe nonlinear characteristics and do not respond continuously in time. In this paper, the stability of discrete-event control systems are analyzed using multiple metrics, simultaneous inequalities, and nonnegative and permutation matrices [4], [5], [6], [7], [8]. As a result, a stability condition is derived based on the concept of nonnegative inverse matrices (so-called M-matrices [9]).

II. Discrete Event Systems and State Transition

In general, finite-state and discrete-event control systems can be written as:

$$\boldsymbol{x}(t_{k+1}) = \boldsymbol{f}(\boldsymbol{x}(t_k), \boldsymbol{e}(t_k)) \qquad (1)$$
$$k \in \mathbb{N} := \{0, 1, 2, \ldots, N\},$$

where $\boldsymbol{x}(\cdot)$, $\boldsymbol{e}(\cdot)$, and $\boldsymbol{f}(\cdot,\cdot)$ are states, event-signals, and a transition function, respectively, as written below:

$$\boldsymbol{x}(t_k) \in \mathbb{Z}^n, \quad \boldsymbol{e}(t_k) \in \mathbb{Z}^m, \boldsymbol{f}: \ \mathbb{Z}^n \times \mathbb{Z}^m \to \mathbb{Z}^n.$$

Although $\mathbb{Z}$ is considered a finite set of integers, we can also define the following expression with resolution value γ:

$$\mathbb{Z}_\gamma := \{-N\gamma, \cdots, -2\gamma, -\gamma, 0, \gamma, 2\gamma, \cdots, N\gamma\}.$$

Here, $\mathbb{Z}_{\gamma+}$ denotes its positive area, and $\mathbb{Z}_1 = \mathbb{Z}$, $\mathbb{Z}_+ = \mathbb{N}$.

Yoshifumi Okuyama is a director of Humanitech Laboratory Co., Ltd., 115-7, Nakatsuura, Hachiman-cho, Tokushima, 770-8072, Japan oku@humanitech-lab.jp

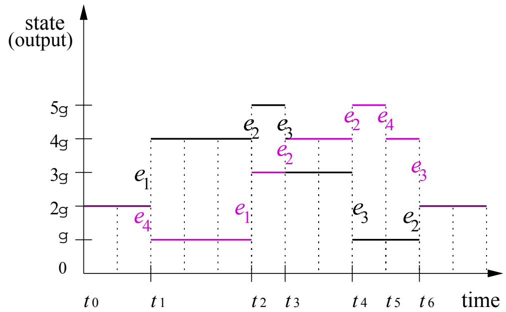


Fig. 1. State trajectories of a discrete event system.

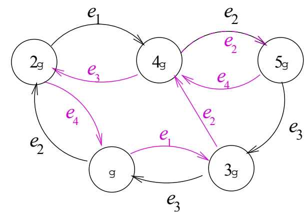


Fig. 2. State transition graph for a discrete event system.

Figure 1 shows an example of state (or output) trajectory for an event-sequence $\{e_1e_2e_3e_3e_2\cdots\}$ (& $e_4e_1e_2e_2e_4e_3\cdots$). As is obvious, it can be considered that the event sequence corresponds to the time sequence $\{t_1t_2t_3t_4t_5t_6\cdots\}$. However, the causality relationship between them will be opposite. In addition, they are not one-to-one correspondence.

In this paper, in order to study the relative stability problem of event-driven control systems [14], [17], we consider the following form:

$$\boldsymbol{x}(t_{k+1}) = \boldsymbol{\mathcal{P}}(t_k)\boldsymbol{x}(t_k) + \boldsymbol{\mathcal{G}}(\boldsymbol{e}(t_k))\boldsymbol{x}(t_k). \qquad (2)$$

Here, $\boldsymbol{\mathcal{P}}(\cdot)$ and $\boldsymbol{\mathcal{G}}(\cdot)$ are assumed to be piecewise constant matrices for $t_k \le t < t_{k+1}$. When corresponding the matrices to (1), the second term of (2) is written as

$$\boldsymbol{\mathcal{G}}(\boldsymbol{e}(t_k))\boldsymbol{x}(t_k) = \boldsymbol{f}(\boldsymbol{x}(t_k), \boldsymbol{e}(t_k)) - \boldsymbol{\mathcal{P}}(t_k)\boldsymbol{x}(t_k). \qquad (3)$$

If we consider simply $\boldsymbol{\mathcal{E}}(t_k) := \boldsymbol{\mathcal{G}}(\boldsymbol{e}(t_k))$, the above expressions can also be written as

$$\boldsymbol{x}(t_{k+1}) = \boldsymbol{\mathcal{P}}(t_k)\boldsymbol{x}(t_k) + \boldsymbol{\mathcal{E}}(t_k)\boldsymbol{x}(t_k), \qquad (4)$$

where

$$\boldsymbol{\mathcal{E}}(t_k) = \begin{bmatrix} \varepsilon_{11}(t_k) & \ldots & \varepsilon_{1n}(t_k) \\ \vdots & \ddots & \vdots \\ \varepsilon_{n1}(t_k) & \ldots & \varepsilon_{nn}(t_k) \end{bmatrix}. \qquad (5)$$

Here, each element is given by

$$\varepsilon_{ij}(t_k) = \left(f_i(\boldsymbol{x}(t_k), \boldsymbol{e}(t_k)) - \sum_{l=1}^{n} \varepsilon_{il} x_l(t_k) \right) / x_j(t_k). \quad (6)$$

Therefore, (2) can be rewritten as

$$\boldsymbol{x}(t_k) = \left(\prod_{h=0}^{k-1} \boldsymbol{\mathcal{P}}(t_h) \right) \boldsymbol{x}(t_0) + \sum_{l=1}^{k} \left(\prod_{h=l}^{k-1} \boldsymbol{\mathcal{P}}(t_h) \right) \boldsymbol{\mathcal{E}}(t_{l-1}) \boldsymbol{x}(t_{l-1}). \quad (7)$$

In the case of "packet losses" and/or "unexpected delays", the above expression will also be applied to the stability and security of networked control systems [10], [11], [12].

III. Connection Matrices and Graphs

When considering the structural properties of systems, we define the following nonnegative constant matrix:

$$\boldsymbol{\mathcal{P}}(t_k) \in \mathbb{I}_+ \subseteq \mathbb{Z}_+^{n \times n}, \qquad \mathbb{I}_+ := \{0, 1\}. \quad (8)$$

Here, a matrix each of whose entries is either 0 or 1 is called a (0,1)-matrix [5].

Directed graphs and cycles. The pair $\boldsymbol{G} = (\boldsymbol{V}, \boldsymbol{E})$ is called a *directed graph*. Here, the elements of $\boldsymbol{V}$ are its *vertices*, and the elements of $\boldsymbol{E}$ are the *arcs* (or *edges*) of $\boldsymbol{G}$. The arc (i, j) is said to *join* vertex i to vertex j. A sequence of arcs $(i, t_1), (t_1, t_2), ..., (t_m, j)$ in $\boldsymbol{G}$ is called a *path* connecting i to j.

The length of the path is defined to be the number m of arcs in the sequence. A path of length m connecting vertex i to itself is called a *cycle* of length m. If each vertex in a cycle appears exactly once as the first vertex of an arc, the cycle is called a *circuit*. A cycle of length 1 is a *self-loop*.

Adjacency and permutation matrices. The *adjacency* matrix $\boldsymbol{A}$ of a directed graph $\boldsymbol{G}$ with n vertices is the (0,1)-matrix (also called *connection* matrix) whose (i, j) entry is 1 if and only if (i, j) is an arc of $\boldsymbol{G}$. Especially, a square matrix that has exactly one 1 in each row and column and 0's elsewhere is called a *permutation* matrix [6], [7], [8].

With respect to a reccurent equation (4), consider the following discrete event systems for $n \geq 3$:

$$\boldsymbol{x}(t_{k+1}) = \boldsymbol{\mathcal{C}}_p \boldsymbol{x}(t_k) + \boldsymbol{\mathcal{E}}(t_h) \boldsymbol{x}(t_k), \quad (9)$$

$$k = 0, 1, 2, \cdots, \quad k \leq h < k+1 \quad (10)$$

where $\boldsymbol{\mathcal{C}}_p$ ($p = 1, 2 \cdots, (n-1)!$) are cycles for n vertices. That is, the nominal system is constant and cyclic. Here, $\boldsymbol{\mathcal{E}}(t_k)$ is a kind of distubance written below:

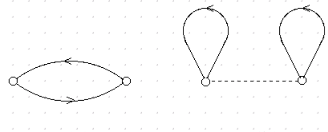
$$\boldsymbol{\mathcal{E}}(t_h) = \boldsymbol{\mathcal{C}}_q(t_h) - \boldsymbol{\mathcal{C}}_p, \quad (p \neq q).$$

In this paper, using such structural matrices, (2) and (3) can be written as follows:

$$\boldsymbol{x}(t_k) = \boldsymbol{\mathcal{C}}_p^k \boldsymbol{x}(t_0) + \sum_{l=1}^{k} \boldsymbol{\mathcal{C}}_p^{k-l} \boldsymbol{\mathcal{E}}(t_{l-1}) \boldsymbol{x}(t_{l-1}). \quad (11)$$

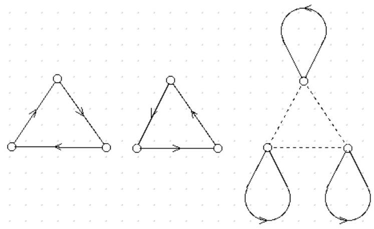
Here, we consider cyclic (periodic) nominal systems. As for third-order periodic systems, the permutation matrices are given by

$$\boldsymbol{\mathcal{C}}_1 = \begin{bmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}, \qquad \boldsymbol{\mathcal{C}}_2 = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{bmatrix},$$



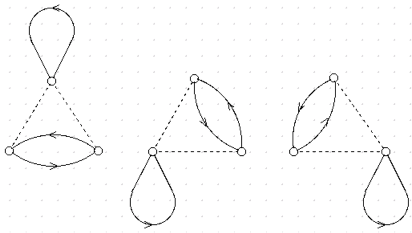
(a) (b)

Fig. 3. Cycles for $n = 2$.



(a) (b) (c)

Fig. 4. Cycles for $n = 3$ (1).



(a) (b) (c)

Fig. 5. Cycles for $n = 3$ (2).

and the directed graphs are as shown in Fig. 4 (a) and (b)[1]. If self-loops (or separate-cycles) are permited, the (0,1)-matrices are given by

$$\boldsymbol{\mathcal{S}}_1 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}, \quad \boldsymbol{\mathcal{S}}_2 = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix}, \quad \boldsymbol{\mathcal{S}}_3 = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix},$$

and the directed graphs are as shown in Fig. 5.

Furthermore, as for fourth-order periodic systems, the permutation matrices are given by

$$\boldsymbol{\mathcal{P}}_{41} = \begin{bmatrix} 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}, \boldsymbol{\mathcal{P}}_{42} = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 \end{bmatrix}, \boldsymbol{\mathcal{P}}_{43} = \begin{bmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}$$

$$\boldsymbol{\mathcal{P}}_{41}^T = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \end{bmatrix}, \boldsymbol{\mathcal{P}}_{42}^T = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}, \boldsymbol{\mathcal{P}}_{43}^T = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{bmatrix}$$

and the directed graphs are as shown in Figs. 6 and 7. Figure 8 shows examples of the permutation graph in which self-loops or or separate-cycles are permitted.

In general, there are $(n-1)!$ permutations with respect to n vertices (i.e., $(n-1)!$ directed graphs (n-cycle digraphs)). If self-loops or separate-cycles are permitted, n directed graphs can be obtained. Figure 9 shows examples of the permutation graphs for $n = 5$.

IV. Multiple Metrics and Inequalities

The metric in the state space (i.e., vector space) is usually defined by a scalar value. However, it may lead to a severe

[1]For reference, cycles of the directed graph for $n = 2$ are given as shown in Fig. 3 (a) and (b). In these graphs, the dotted lines are hypothetical edges.

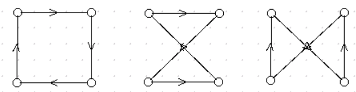


Fig. 6. Cycles for $n = 4$ (1).

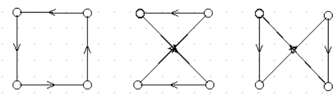


Fig. 7. Cycles for $n = 4$ (2).

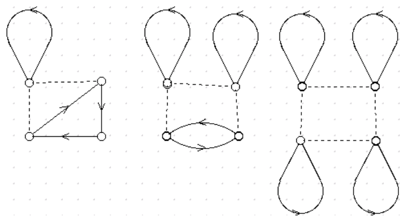


Fig. 8. Cycles for $n = 4$ (3).

condition for the stability of some kind of nonlinear systems. Therefore, we consider the metric (i.e., ℓ_∞-norm) for each element of the state as follows:

$$\|x_i(t_k)\|_{\ell_\infty} := \sup_{1 \le l \le k} |x_i(t_l)| \in \mathbb{Z}_+. \tag{12}$$

Furthermore, in this paper, we can define a new metric (i.e., ℓ_1-norm),

$$\|x_i(t_k)\|_{\ell_1} = \sum_{l=1}^{k} |x(t_l)| \in \mathbb{Z}_+. \tag{13}$$

When considering multiple metrics, the following vector expression can be defined:

$$\|\boldsymbol{x}(t_k)\| = \begin{bmatrix} \|x_1(t_k)\| \\ \|x_2(t_k)\| \\ \vdots \\ \|x_n(t_k)\| \end{bmatrix} \in \mathbb{Z}_+^n, \tag{14}$$

where $\|x_i(t_k)\|$ are $\|x_i(t_k)\|_{\ell_\infty}$ or $\|x_i(t_k)\|_{\ell_1}$ [2].

Based on these considerations, the following inequalities are obtained from (11) [3]:

$$\|\boldsymbol{x}(t_k)\| \leqq \|\bar{\boldsymbol{x}}(t_k)\| + \left\| \sum_{l=1}^{k} \boldsymbol{\Psi}(t_k, t_l)\boldsymbol{x}(t_{l-1}) \right\|, \tag{15}$$

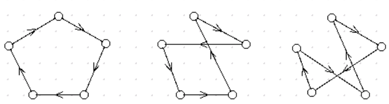


Fig. 9. Examples of cycles for $n = 5$.

where $\bar{\boldsymbol{x}}(t_k)$ is the nominal system, i.e.,

$$\bar{\boldsymbol{x}}(t_k) = \boldsymbol{\mathcal{C}}_p^k \boldsymbol{x}(t_0)$$

and $\boldsymbol{\Psi}(t_k, t_l)$ is a new transition matrix in (11) as follows:

$$\boldsymbol{\Psi}(t_k, t_l) = \boldsymbol{\mathcal{C}}_p^k \boldsymbol{\mathcal{E}}(t_{l-1}).$$

Here, we consider the following nonnegative matrix:

$$\|\boldsymbol{\Theta}(t_k)\| := \begin{bmatrix} \|\theta_{11}(t_k)\| & \dots & \|\theta_{1n}(t_k)\| \\ \vdots & \ddots & \vdots \\ \|\theta_{n1}(t_k)\| & \dots & \|\theta_{nn}(t_k)\| \end{bmatrix},$$

where,

$$\|\theta_{ij}(t_k)\| = \left\| \sum_{l=1}^{k} \psi_{ij}(t_k, t_l) x_j(t_{l-1}) \right\| / \|x_j(t_k)\|$$
$$\|\theta_{ij}(t_k)\| \in \mathbb{R}_+ \quad i, j = 1, 2, \cdots, n.$$

Therefore, inequality (15) is written as

$$\|\boldsymbol{x}(t_k)\| \leqq \|\bar{\boldsymbol{x}}(t_k)\| + \|\boldsymbol{\Theta}(t_k)\| \cdot \|\boldsymbol{x}(t_k)\|. \tag{16}$$

Moreover, it can be given as follows:

$$\left(\boldsymbol{I} - \left\|\boldsymbol{\Theta}(t_k)\right\|\right) \|\boldsymbol{x}(t_k)\| \leqq \|\bar{\boldsymbol{x}}(t_k)\|. \tag{17}$$

Here, we write the above inequality as:

$$(\boldsymbol{I} - \boldsymbol{C})\boldsymbol{X} \leqq \boldsymbol{Y}, \tag{18}$$

where $\boldsymbol{C} \geqq \boldsymbol{0}$, $\boldsymbol{X} \geqq \boldsymbol{0}$, and $\boldsymbol{Y} \geqq \boldsymbol{0}$ correspond to $\|\boldsymbol{\Theta}(t_k)\|$, $\|\boldsymbol{x}(t_k)\|$, and $\|\bar{\boldsymbol{x}}(t_k)\|$, respectively.

V. Nonnegative Inverse Matrices

Consider the following square matrix:

$$\boldsymbol{A} = \begin{bmatrix} a_{11} & -a_{12} & \dots & -a_{1n} \\ -a_{21} & a_{22} & \dots & -a_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ -a_{n1} & -a_{n2} & \dots & a_{nn} \end{bmatrix}, \tag{19}$$

where $a_{ij} \geq 0$ $(i, j = 1, 2, \cdots, n)$. It is said that the most common situation in the biological, physical, and social sciences is where the matrix $\boldsymbol{A}$ has nonpositive off-diagonal and nonnegative diagonal entries [4]. Of course, it can simply be written as $\boldsymbol{A} = [a_{i,j}]$. However, in this paper, the expression of (19) will be used in order to clarify the sign of each element.

If we apply $\boldsymbol{A}$ to $\boldsymbol{I} - \boldsymbol{C}$ in (18), with respect to

$$\boldsymbol{C} = \begin{bmatrix} c_{11} & \dots & c_{1n} \\ \vdots & \ddots & \dots \\ c_{n1} & \dots & c_{nn} \end{bmatrix} \tag{20}$$

$1 - c_{ii} \geq 0$ and $c_{ij} \geq 0$ $(i \neq j)$ are obtained, though $1 - c_{ii} > 0$ is considered in the engineering applications.

Lemma. For any $\boldsymbol{0} \leqq \boldsymbol{Y} < \infty$, we can obtain $\boldsymbol{0} \leqq \boldsymbol{X} < \infty$ if and only if $\boldsymbol{A} = \boldsymbol{I} - \boldsymbol{C}$ is a nonnegative-inverse matrix [13].

Proof. The proof is obtained from the property of nonnegative-inverse matrix (i.e., $\boldsymbol{A}^{-1} \geqq \boldsymbol{0}$). □

[2]Hereafter, each of them is written simply as $\|\cdot\|$. However, in the following examples, only $\|\cdot\|_{\ell_\infty}$ will be considered.

[3]Inequality symbols for matrices and vectors are based on [13]

Based on the above, the stability (i.e., boundednes) conditions is given as follows.

Definition. If $\|\bar{\boldsymbol{x}}(t_k)\| \leqq \boldsymbol{Y}$ leads to $\|\boldsymbol{x}(t_k)\| \leqq \boldsymbol{X}$ for all $k \in \mathbb{N}$, the discrete event system is defined as (finite-time) stable in a relative sense [14]. Here, $\boldsymbol{X}$ and $\boldsymbol{Y}$ are vectors of some finite (positive) numbers. □

Thus, the following theorem is given.

Theorem. If $\boldsymbol{A} = \boldsymbol{I} - \boldsymbol{C} = \boldsymbol{I} - \|\boldsymbol{\Theta}(t_k)\|$ is a nonnegative-inverse matrix (i.e., Ostrowski's M-matrix), the system is stable (and bounded) in a relative sense. That is, a finite $\boldsymbol{X} \in \mathbb{Z}_+^n$ can be obtained for any $\boldsymbol{Y} \in \mathbb{Z}_+^n$.

Proof. The condions of nonnegative-inverse matrix for $\boldsymbol{A} = \boldsymbol{I} - \boldsymbol{C}$ are given as follows.

(1) $\rho(\boldsymbol{C}) := \max_{1 \le i \le n} |\lambda_i| < 1$, where λ_i are eigenvalues of $\boldsymbol{C}$.

(2) The principal minors of $\boldsymbol{A}$ are all positive (i.e., $\Delta_i > 0, \quad 1 \le i \le n$).

In this paper, we will prove the above (2). The simultaneous inequality (18) is written as

$$\begin{bmatrix} a_{11} & -a_{12} & \dots & -a_{1n} \\ -a_{21} & a_{22} & \dots & -a_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ -a_{n1} & -a_{n2} & \dots & a_{nn} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_3 \end{bmatrix} \leqq \begin{bmatrix} y_1 \\ y_2 \\ \vdots \\ y_3 \end{bmatrix}. \tag{21}$$

By using an elimination method, the above inequality can be rewritten as follows:

$$\begin{bmatrix} a_{11}^{(1)} & -a_{12}^{(1)} & \dots & -a_{1n}^{(1)} \\ 0 & a_{22}^{(2)} & \dots & -a_{2n}^{(2)} \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \dots & a_{nn}^{(n)} \end{bmatrix} \begin{bmatrix} x_1^{(1)} \\ x_2^{(1)} \\ \vdots \\ x_n^{(1)} \end{bmatrix} \leqq \begin{bmatrix} y_1^{(1)} \\ y_2^{(2)} \\ \vdots \\ y_n^{(n)} \end{bmatrix}, \tag{22}$$

where

$$a_{ij}^{(1)} = a_{ij}, \quad x_j^{(1)} = x_j, \quad y_i^{(1)} = y_i \ (i,j = 1,2,\cdots,n).$$

Furthermore,

$$\begin{cases} a_{ii}^{(2)} = \dfrac{1}{a_{11}^{(1)}} \begin{vmatrix} a_{11}^{(1)} & -a_{1i}^{(1)} \\ -a_{i1}^{(1)} & a_{ii}^{(1)} \end{vmatrix}, \quad a_{ij}^{(2)} = \dfrac{1}{a_{11}^{(1)}} \begin{vmatrix} a_{11}^{(1)} & -a_{1j}^{(1)} \\ -a_{i1}^{(1)} & -a_{ij}^{(1)} \end{vmatrix}, \\ \qquad (i,j = 2,3) \\ \vdots \\ a_{nn}^{(n)} = \dfrac{1}{a_{n-1\ n-1}^{(n-1)}} \begin{vmatrix} a_{n-1\ n-1}^{(n-1)} & -a_{n-1\ n}^{(n-1)} \\ -a_{n\ n-1}^{(n-1)} & a_{nn}^{(n-1)} \end{vmatrix}. \end{cases}$$

Therefore, the right side of (17) can be written as

$$y_1^{(1)} = y_1, \ y_2^{(2)} = y_2^{(1)} + \frac{a_{21}^{(1)}}{a_{11}^{(1)}} y_1^{(1)}, \ y_3^{(3)} = y_3^{(2)} + \frac{a_{32}^{(2)}}{a_{22}^{(2)}} y_2^{(2)},$$
$$\cdots\cdots, \ y_n^{(n)} = y_n^{(n-1)} + \frac{a_{n\ n-1}^{(n-1)}}{a_{n-1\ n-1}^{(n-1)}} y_{n-1}^{(n-1)},$$

provided $a_{11}^{(1)} > 0$, $a_{22}^{(2)} > 0$, $\cdots$, $a_{n-1\ n-1}^{(n-1)} > 0$. It can be seen that these values are non-negative and bounded if each vector y_i is bounded (i.e., $y_i^{(1)} < \infty, \ i = 1,2,\cdots,n$). In addition, if $a_{nn}^{(n)} > 0$ is satisfied, then $x_n^{(1)} < \infty$, $x_{n-1}^{(1)} < \infty$, $\cdots$, and $x_1^{(1)} < \infty$ are obtained in reverse order.

Here, it should be noted that the above conditions $a_{11}^{(1)} > 0$, $a_{22}^{(2)} > 0$, $a_{33}^{(3)} > 0$, $\cdots$, $a_{nn}^{(n)}$ are rewritten as follows:

$$a_{11}^{(1)} = \Delta_1 = a_{11} > 0$$
$$a_{22}^{(2)} = \frac{\Delta_2}{\Delta_1} = \begin{vmatrix} a_{11} & -a_{12} \\ -a_{21} & a_{22} \end{vmatrix} \Big/ a_{11} > 0$$
$$a_{33}^{(3)} = \frac{\Delta_3}{\Delta_2} = \begin{vmatrix} a_{11} & -a_{12} & -a_{13} \\ -a_{21} & a_{22} & -a_{23} \\ -a_{31} & -a_{32} & a_{33} \end{vmatrix} \Big/ \begin{vmatrix} a_{11} & -a_{12} \\ -a_{21} & a_{22} \end{vmatrix} > 0$$
$$\vdots$$
$$a_{nn}^{(n)} = \frac{\Delta_n}{\Delta_{n-1}} =$$
$$\begin{vmatrix} a_{11} & -a_{12} & \dots & -a_{1n} \\ -a_{21} & a_{22} & \dots & -a_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ -a_{n1} & -a_{n2} & \dots & a_{nn} \end{vmatrix} \Big/ \begin{vmatrix} a_{11} & \dots & -a_{1\ n-1} \\ \vdots & \ddots & \vdots \\ -a_{n-1\ 1} & \dots & a_{n-1\ n-1} \end{vmatrix} > 0.$$

These conditions say that all principal minors of matrix $\boldsymbol{A}$ are positive (i.e., $\Delta_i > 0, \quad 1 \le i \le n$) [15], [16], [17]. It means that the matrix becomes a nonegative-inverse matrix, that is, Ostrowski's M-matrix.

VI. Numerical Examples

Example 1. Consider the following recurrent third-order system disturbed by some uncertain event-signals:

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}_{k+1} = \begin{bmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}_k + \begin{bmatrix} 0 & 0 & \epsilon_{13} \\ \epsilon_{21} & 0 & 0 \\ 0 & \epsilon_{32} & 0 \end{bmatrix}_k \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}_k.$$

The nominal system of this example is periodic with a period $n = 3$ as shown in Fig. 4 (a). Here, we assume that disturbed event-signals $\epsilon_{13} = 0.1$, $\epsilon_{21} = -0.1$, and $\epsilon_{32} = 0.05$ are inserted periodically as shown in Fig. 10. Figure 11 shows state traces from $x_1(0) = -2.0$, $x_2(0) = 2.0$, $x_3(0) = 1.0$ for $t_k \le 200$. The state trace representation in the 3D coordinates is given as shown in Fig. 12. The response will be pseudo-periodic and obviously bounded (i.e., relatively stable) for $t_k \le 200$. Figure 13 shows the time-sequences of Δ_i $(i = 1,2,3)$. From this figure, the stabilty condition will be satisfied at least in the area, $t_k \le 150$.

On the other hand, Fig. 14 shows the case where the disturbed signals are given as $\epsilon_{13} = 0.2$, $\epsilon_{21} = -0.2$, and $\epsilon_{32} = 0.1$. Figure 15 shows state traces from $x_1(0) = -2.0$,

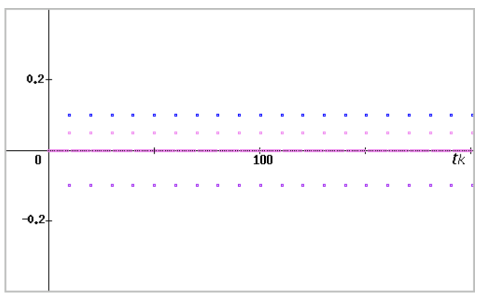


Fig. 10. Time series of event signals; peak values $\epsilon_{13} = 0.1$, $\epsilon_{21} = -0.1$, and $\epsilon_{32} = 0.05$.

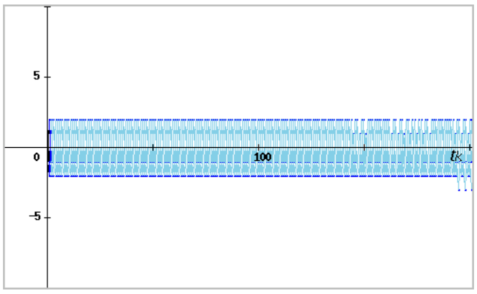


Fig. 11. State traces when $x_1(0) = -2.0$, $x_2(0) = 2.0$, and $x_3(0) = 1.0$.

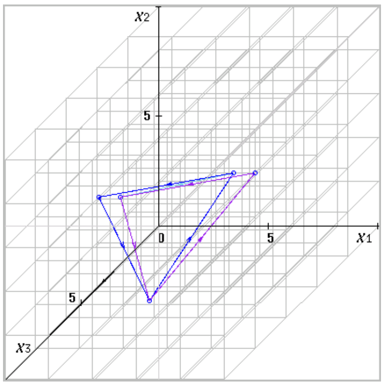


Fig. 12. A state trace in the 3D coordinates ($t_k \leq 200$).

$x_2(0) = 2.0$, $x_3(0) = 1.0$ (same as the above) for $t_k \leq 200$. In this case, the state trace representation in the 3D coordinates is given as shown in Fig. 16. The response becomes not periodic but rather chaotic. Figure 17 shows the time-sequences of $\varDelta_i$ $(i = 1, 2, 3)$. From these figures, the stabilty condition will not be satisfied in most areas.

Example 2. In this example, we consider the cases where some connections of the discrete-event system are failed as shown in Fig. 18. Figure 19 shows the state trace in the 3D coordinates when only the first failure ϵ_{13} is occured. In this case, the trajectory will be settled into an another periodic one (a triangle).

On the other hand, when three failures are occured as shown in Fig. 18, the trajectories may diverge. Figure 20 shows the time series of state traces. The state trace in the 3D coordinates is given as shown in Fig. 21. In this case, the stability will not be guranteed. The time-sequences of $\varDelta_i$ $(i = 1, 2, 3)$ become as shown in Fig. 22.

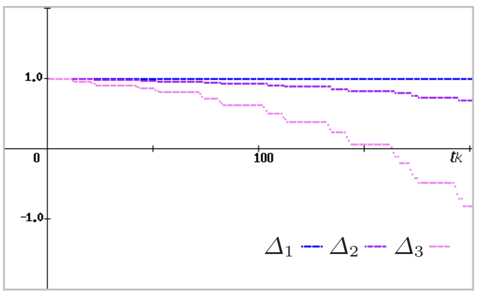


Fig. 13. Time series of $\varDelta_1$, $\varDelta_2$, and $\varDelta_3$.

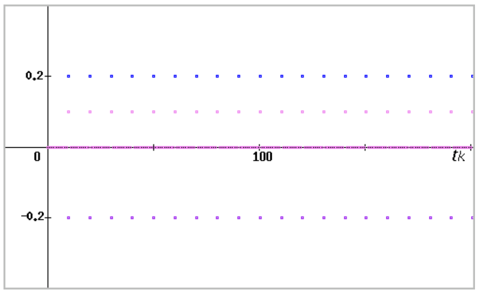


Fig. 14. Time series of event signals; peak values $\epsilon_{13} = 0.2$, $\epsilon_{21} = -0.2$, and $\epsilon_{32} = 0.1$.

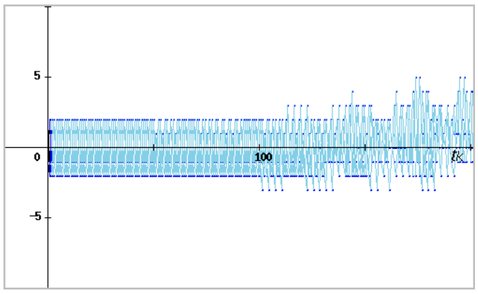


Fig. 15. State traces when $x_1(0) = -2.0$, $x_2(0) = 2.0$, and $x_3(0) = 1.0$.

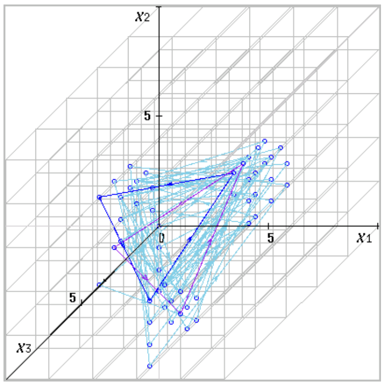


Fig. 16. A state trace in the 3D coordinates ($t_k \leq 200$).

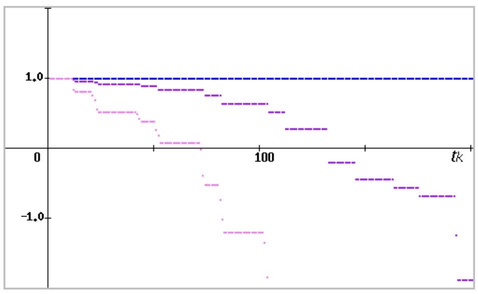


Fig. 17. Time series of $\varDelta_1$, $\varDelta_2$, and $\varDelta_3$.

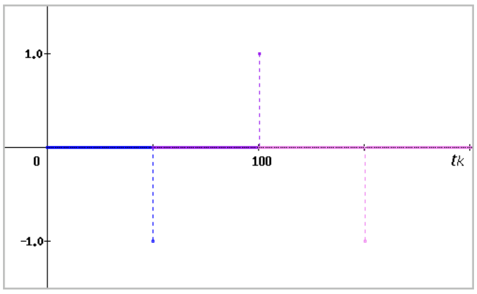


Fig. 18. Examples of event signals; peak values $\epsilon_{13} = -1.0$ or $\{\epsilon_{13} = -1.0,\ \epsilon_{21} = 1.0$ and $\epsilon_{32} = -1.0\}$.

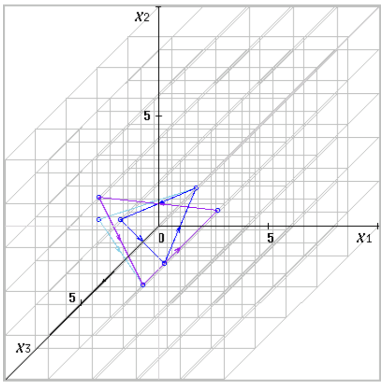


Fig. 19. A state trace in the 3D coordinates $(t_k \leq 200)$.

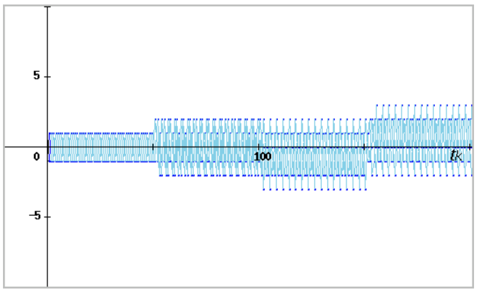


Fig. 20. State traces when $x_1(0) = -1.0$, $x_2(0) = 1.0$, and $x_3(0) = 1.0$.

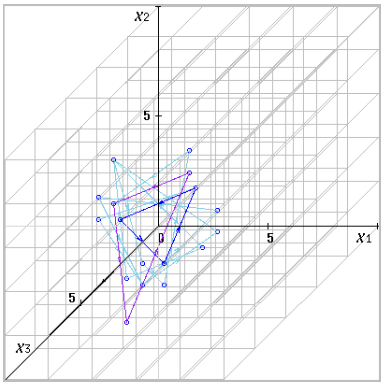


Fig. 21. A state trace in the 3D coordinates $(t_k \leq 200)$.

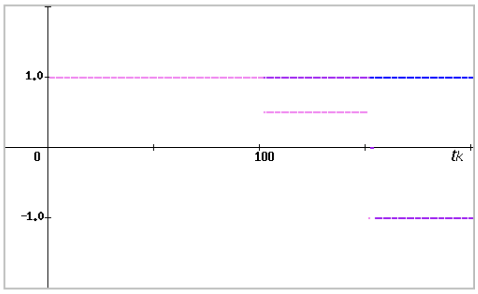


Fig. 22. Time series of Δ_1, Δ_2, and Δ_3.

VII. Conclusions

The stability of discrete-event control systems has been studied using multiple metrics and simultaneous inequalities. Especially, in this paper, the structures of discrete even systems were analyzed based on connection matrices (i.e., permutation (0,1)-matrices). The relationship between connection matrices and graph representations was also reviewed in general. As a result, a stability condition was derived based on the concept of nonnegative inverse matrices (so-called M-matrices). Numerical examples were shown to clarify the stability and boundedness of discrete-event control systems. The result will be useful for the analysis and design of discrete control systems in general.

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